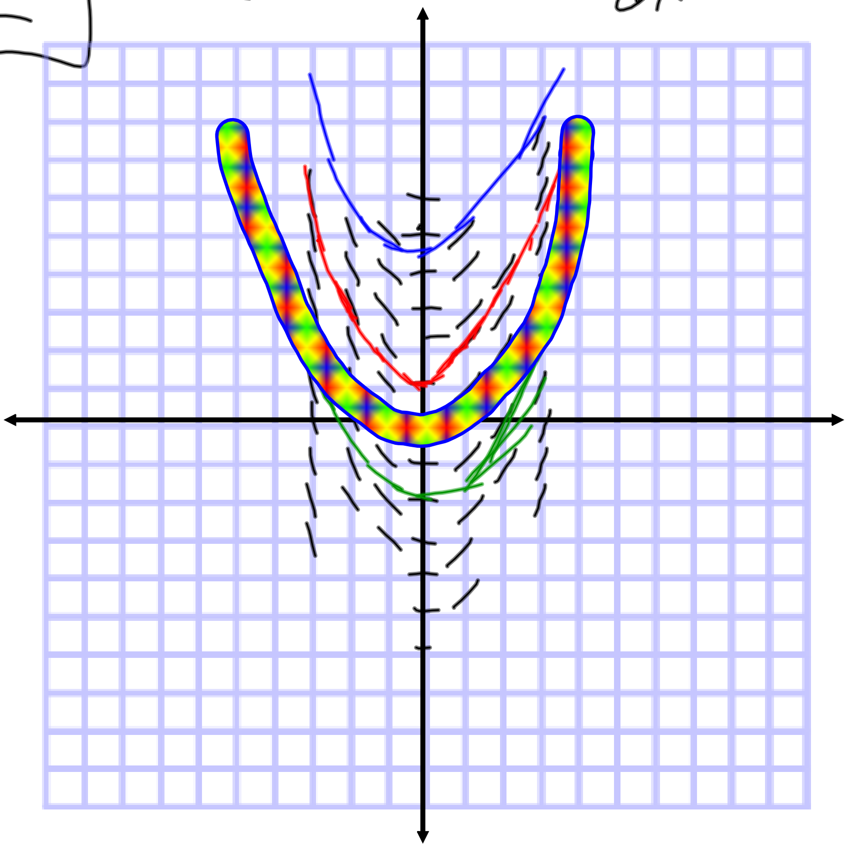
6.2/35

a) slope field for $\frac{dy}{dx} = x$; $x, y \in [-5, 5]$



6.2/41

a) $\frac{dy}{dx} = \sqrt[3]{x}$; $y(1) = 2$

$$\frac{dy}{dx} = x^{1/3}$$

step 1
find $\int \underline{dx}$

$$\int x^{1/3} dx = \frac{x^{4/3}}{4/3} + C$$

$$y = \frac{3x^{4/3}}{4} + C$$

$$2 = \frac{3(1)^{4/3}}{4} + C$$

$$2 = \frac{3}{4} + C \quad \therefore C = \frac{5}{4}$$

$$y = \frac{3x^{4/3}}{4} + \frac{5}{4}$$

6.2/42c

$\frac{dy}{dx} = x^2\sqrt{x^3}$; $y(0) = 0$

$(x^{\frac{1}{2}})^3$ or $(x^3)^{1/2}$

$y = \int x^2\sqrt{x^3}\,dx$

$= \int x^2 (x^{\frac{3}{2}})\,dx = \int x^{\frac{7}{2}}\,dx$

$y = \frac{x^{\frac{9}{2}}}{9/2} + C$ or $y = \frac{2x^{\frac{9}{2}}}{9} + C$

step 2
initial
conditi

$y(0) = 0$

$0 = \frac{2}{9}(0) + C \therefore C = 0$

$y = \frac{2x^{9/2}}{9}$

$$\int \frac{x+1}{\sqrt{x}}\,dx = \int \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\,dx \cdots$$

2

6.3 Integration by substitution

AKA ... attempt at "un" chain rule

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$$

$$\int f'(g(x))\, g'(x)\, dx = f(g(x)) + C$$

$$\int \underbrace{(x^2+1)^{50}}_{}\ \boxed{2x\,dx} = \int u^{50}\,du$$

$f'(g(x))$ $g'(x)$

Let $\boxed{u = x^2+1}$

then $\frac{du}{dx} = 2x$

and $\boxed{du = 2x\,dx}$

$$= \frac{u^{51}}{51} + C$$

$$= \frac{(x^2+1)^{51}}{51} + C$$

$$\frac{d}{dx}\left(\frac{(x^2+1)^{51}}{51} + C\right) = (x^2+1)^{50} \cdot 2x$$

$$\int \sin(x+9)\, dx \qquad \int \sin u \, du$$

Let $u = x+9$

$\frac{du}{dx} = 1$

$du = dx$

$$\int \sin u \quad du$$

$= -\cos u + C$

$= -\cos(x+9) + C$

$$\int \overset{u}{(4-3x)}^{22}\,\overset{du}{dx} \Rightarrow \int u^{22}\,du$$

$$u = 4-3x$$

$$\frac{du}{dx} = -3$$

$$\frac{u^{23}}{23} + C$$

$$\int -\frac{1}{3}\underbrace{(4-3x)}_{u}^{22}\underbrace{(-3)}_{u'}dx$$

$$\int -\frac{1}{3}(u)^{22}du$$

$$= -\frac{u^{23}}{69} + C$$

$$u = 4-3x$$

$$\frac{du}{dx} = -3$$

$$du = \boxed{-3dx}$$

$$= \frac{(4-3x)^{23}}{69} + C$$

$$\int (4-3x)^{22}\,dx$$

$$u = 4-3x$$

$$\frac{du}{dx} = -3$$

$$\frac{du}{-3} = \frac{-3\,dx}{-3}$$

$$-\frac{1}{3}du = dx$$

$$\int u^{22}\left(-\frac{1}{3}\right)du$$

$$= -\frac{1}{3}\int u^{22}\,du$$

$$\int \cos(5x)\,dx$$

want $\int \cos u\,du$ so

Let $u = 5x$

$$\frac{du}{dx} = 5$$

$$du = 5\,dx$$

$$\frac{1}{5}du = dx$$

$$\int \cos u \left(\frac{1}{5}\right) du =$$

$$\frac{1}{5}\int \cos u\,du = \frac{1}{5}(\sin u) + C$$

$$= \frac{1}{5}\sin(5x) + C$$

$$(3)\int \frac{\left(\frac{1}{3}\right)dx}{\left(\frac{1}{3}x - 8\right)^5}$$

$$u = \frac{1}{3}x - 8$$

$$\frac{du}{dx} = \frac{1}{3}$$

$$du = \frac{1}{3}dx$$

$$3\int \frac{du}{u^5}$$

$$= 3\int u^{-5}\,du$$

$$= 3\left(\frac{u^{-4}}{-4}\right) + C$$

$$= -\frac{3}{4}\left(\frac{1}{3}x - 8\right)^{-4} + C$$

$\int \sin^2 x \cos x \, dx$

$(\sin x)(\sin x)(\cos x)\, dx$

$u = \sin^2 x$

$\frac{du}{dx} = 2(\sin x)\cos x$

$\frac{1}{2} du = \sin x \cos x \, dx$

$\frac{1}{2}\int \sqrt{u} \, du = \frac{1}{2}\int u^{1/2} du$

$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{1}{3}(\sin^2 x)^{3/2} + C$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$= \int u^2 du = \frac{u^3}{3} + C = \frac{\sin^3 x}{3} + C$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$-du = \sin x \, dx$

$\sin x \cos x$

$-\int \sqrt{1-u^2}\, u \, du$

$\sin^2 x = 1 - \cos^2 x$

$\sin x = \sqrt{1-\cos^2 x}$

Let $v = 1 - u^2$

$\frac{dv}{du} = -2u$

$\vdots$

$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

$\int e^u du$

$u = \sqrt{x} = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$2du = \frac{1}{\sqrt{x}} dx$

$2\int e^u \, du$

$\vdots$

6.2/35

$\frac{dy}{dx} = x$

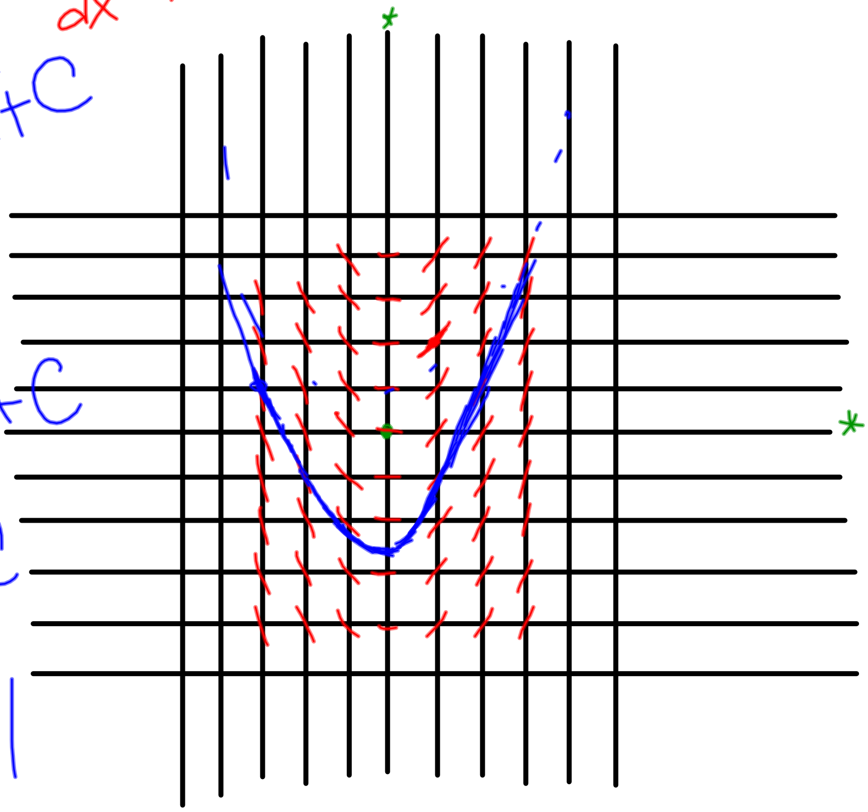
$y = \frac{1}{2}x^2 + C$

$(4,7)$

$7 = \frac{1}{2}(4)^2 + C$

$-1 = C$

$\therefore y = \frac{1}{2}x^2 - 1$



41) a $\frac{dy}{dx} = \sqrt[3]{x}$; $y(1) = 2$

$$\int x^{\frac{1}{3}} dx$$

$$\frac{x^{4/3}}{4/3} \quad \frac{3x^{4/3}}{4} + C$$

$$\frac{3(1)^{4/3}}{4} + C = 2 - 3/4$$
$$-3/4$$

$$C = \frac{5}{4} = \frac{V}{IV}$$

$$\frac{3x^{4/3}}{4} + \frac{5}{4} = y$$

6.3§ Integration by substitution

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\cdot g'(x)$$

$$\int f'(\boxed{g(x)})\cdot \boxed{g'(x)\,dx} = f(g(x)) + C$$

$$\int (x^2+1)^{50}\, 2x\,dx \qquad \int u^{50}\, du$$

Let $\boxed{u = x^2+1}$

$$= \frac{u^{51}}{51} + C$$

$$\frac{du}{dx} = 2x$$

$$= \frac{(x^2+1)^{51}}{51} + C$$

$$\boxed{du = 2x\,dx}$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \sin(x+9)\,\boxed{dx}$$

Let $u = \underline{x+9}$

$$\frac{du}{dx} = 1$$

$$du = \boxed{dx}$$

$$\int \sin(u)\,du = -\cos(u) + C$$

$$= -\cos(x+9) + C$$

$$\int (4-3x)^{22}\,\boxed{dx}$$

$$\int u^{22}\,du$$

Let $u = 4-3x$

$$\frac{du}{dx} = -3$$

$$\frac{du}{-3} = \frac{-3dx}{-3}$$

$$-\frac{1}{3}du = \boxed{dx}$$

$$\int u^{22}\left(-\frac{1}{3}\right)du$$

$$= -\frac{1}{3}\int u^{22}\,du$$

$$= -\frac{1}{3}\left(\frac{u^{23}}{23}\right) + C$$

$$= -\frac{1}{3}\left(\frac{(4-3x)^{23}}{23}\right) + C$$

$$\int \cos(5x)\,dx$$

$$u = 5x \rightarrow \frac{du}{dx} = 5$$

$$dx = \frac{1}{5}du$$

$$\int \cos(u)\,\frac{du}{5}$$

$$\frac{1}{5}\int \cos u\,(du)$$

$$= \frac{1}{5}\sin(5x) + \text{Baltic}$$

$$\int \frac{dx}{\left(\frac{1}{3}x-8\right)^5}$$

$$\int \frac{1}{\left(\frac{1}{3}x-8\right)^5}\cdot dx = \int \frac{1}{u^5}\,3du$$

☆ $\frac{1}{3}x - 8 = u$ ☆☆

☆ $\frac{1}{3}du = \frac{du}{\cdot 3}$ ☆☆☆

$\cdot 3$

$$du = 3dx$$

$$\int 3\cdot u^{-5}\,du$$

$$3\cdot\frac{u^{-4}}{4} = \frac{3}{-4u^4}$$

$$\frac{3}{-4u^4} + C$$

$$\frac{3}{-4\left(\frac{1}{3}x-8\right)^4} + C$$

$$\int \sin^2 x \boxed{\cos x\, dx}$$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \boxed{\cos x\, dx}$

$$\int u^2\, du = \frac{u^3}{3} + C$$

$$= \frac{\sin^3 x}{3} + C$$

$$\int \sin^2 x \cos x\, dx$$

Let $u = (\sin^2 x)$

$\frac{du}{dx} = 2 \sin x \cos x$

$\frac{1}{2} du = \sin x \cos x\, dx$

$$\frac{1}{2} \int u^{1/2}\, du$$

$$= \frac{1}{2} \frac{u^{3/2}}{3/2} + C = \frac{1}{2} \cdot \frac{2}{3} (\sin^2 x)^{3/2} + C$$

$$\int \sin^2 x \cos x\, dx$$

$u = \cos x$

$-du = +\sin x\, dx$

$\sin x \cos x$

$$-\int \sqrt{1-u^2}\, u\, du$$

$\sin^2 x = 1 - \cos^2 x$

$\sin x = \sqrt{1 - \cos^2 x}$

Let $v = 1 - u^2$

$dv = -2u\, du$

$\vdots$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx \qquad 2 \int e^u\, du$$

$u = \sqrt{x} = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2} x^{-1/2}$

$2\, du = \boxed{\frac{1}{\sqrt{x}} dx}$

$= 2e^u + C$

$= 2e^{\sqrt{x}} + C$

~~Qed~~

QED

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2 \int du$$

Let $u = e^{\sqrt{x}}$

$$\frac{du}{dx} = e^{\sqrt{x}} \left(\frac{1}{2} x^{-1/2} \right)$$

$$2\, du = \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$